ISSN 2723-6471

The Model of Carbon Price Risk Prediction in European Markets Using Long Short-Term Memory- Geometric Brownian Motion

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(Received: November 5, 2024; Revised: December 15, 2024; Accepted: January 17, 2025; Available online: February 21, 2025)

Abstract

Accurate carbon market price prediction is one of the fundamentals in assessing the risks associated with carbon trading. Related studies on carbon price prediction were mainly focused on two major approaches: mathematical and/or machine learning models. Geometric Brownian Motion (GBM) is one of the mathematical models that can represent carbon price movements but requires modifying the sample size and the number of parameters for compiling the simulation numerically. Moreover, two critical parameters: (μ) mu and (σ) sigma need to be estimated to simulate the carbon price movements. In this study, the parameters μ and σ estimation are based on the average return value and standard deviation. However, if the carbon price movement is very volatile, we need to recognize its trend and characteristics by estimating the parameters precisely until there is no significant change (or stable) patterns. That is very expensive and may be intractable on high-dimensional data with less precise prediction. Therefore, we propose a hybrid model for carbon price prediction based on GBM with the parameter estimation using the Long Short-Term Memory (LSTM) model. The LSTM model was chosen because it has high accuracy in parameter estimation without losing the characteristics of the GBM stochastic model. Furthermore, Value at Risk (VaR) is utilized to measure the risk of carbon price volatility predictions. The simulation results showed the proposed model has higher prediction accuracy with a not-too-significant time difference, and the model is proven reliable in measuring future risks.

Keywords: Carbon Price, Prediction, Risk Model Long Short-Term Memory, Stochastic Differential Equation

1. Introduction

Climate change occurs due to greenhouse gas (GHG) emissions from human activities in various sectors. Incidents such as rising sea levels, extreme global warming, forest fires, floods, and other natural phenomena are increasingly occurring worldwide [1]. International efforts to mitigate the effects of climate change rely on global agreements like the United Nations Framework Convention on Climate Change, the Kyoto Protocol, and the Paris Agreement [2]. Carbon dioxide is a gas with the highest emission composition, often called carbon emissions [3], [4]. Most carbon dioxide is produced by industrial and energy sectors [2]. Therefore, companies that emit carbon will be subject to sanctions in the form of carbon emission fees. In contrast, companies that succeed in reducing emissions will be given incentives, from which transactions called carbon markets are created [4]. Increasing and uncontrolled carbon emissions will pose a risk of global warming, so it is necessary to implement environmental policies and regulations that support reducing carbon emissions. The environmental policies under Article 2 of the 2015 Paris Agreement aim to ensure that financial investments support the reduction of greenhouse gas emissions and promote climate-resilient development. Following the Paris Agreement, the government and industry implemented policies, including setting emission quotas and incentives. If the permitted emission quota is reduced, it will cause the carbon price to rise. Conversely, if the permitted emission quota is increased, it will cause the carbon price to fall. So, emission quotas and incentives will influence the carbon price [2].

Accurate predictions of carbon market prices are fundamental for assessing the risks associated with carbon financing policies [5]. Predictions support investors in managing the risk of carbon price fluctuations, preserving and increasing

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DOI: https://doi.org/10.47738/jads.v6i2.536

the value of carbon assets [6]. Understanding carbon pricing trends provides insight into changing corporate and social behavior towards environmentally friendly and low-emission practices. Therefore, carbon market price prediction has become essential in international energy and climate economics research [7]. Research on carbon price prediction includes two main categories: mathematical and statistical models [1], [3], [8], [9], [10] including GBM, Error-Trend-Seasonal (ETS), and machine learning algorithms [1], [11], [12], [13], [14], [15] including Artificial Neural Network (ANN), Least Square Support Vector Machine (LSSVM), LSTM and various ensemble methods. From existing machine learning methods, the hybrid method shows that this approach produces competitive performance compared to other models in the literature [16].

In previous studies, GBM can represent carbon price movement patterns [17]. However, GBM requires sample size and parameter modification to represent carbon price movement patterns in order to perform numerical simulations. In the GBM model, carbon price movements are influenced by two parameters: the parameters μ and σ. These parameters μ and σ greatly affect the pattern and movement of carbon prices. To be able to simulate carbon price movements, estimates of the parameters μ and σ are needed. So far, the estimates of the parameters μ and σ in the GBM model have used the average return value and standard deviation, and this can be done if the carbon price data has a stable pattern and the change pattern is not significant, however, if the carbon price movement is very volatile and has a high volatility value, a parameter estimation model is needed to recognize patterns and characteristics of carbon price movements more precisely [16]. Therefore, we propose measuring carbon price volatility risk based on the results of carbon price forecasting using the hybrid LSTM and GBM methods with Monte Carlo simulations. The study was conducted by estimating the parameters μ and σ of carbon prices using LSTM, then we estimated carbon prices using the GBM method. Furthermore, the most accurate carbon price prediction results are used to predict the risk of carbon price volatility using VaR. Tami [18] suggested that further research could explore using LSTM architecture in predicting commodity prices. By developing a hybrid model that combines the strengths of LSTM with other models, promising results can be achieved.

From the existing references, it is stated that LSTM has proven to be a method that has superior performance in time series forecasting. LSTM has a long-term memory that is more effective because it allows more parameters to be learned [5]. LSTM has the advantage of capturing the characteristics of complex nonlinear series relationships from various data [19]. LSTM is also consistent with its capacity to extract complex nonlinear correlations from financial data. LSTM implies that machine learning (ML) techniques can improve prediction accuracy in various domains [20]. In the context of carbon prices, LSTM offers higher accuracy than other methods and provides better stability [21].

This paper aims to improve the performance of the GBM model in predicting carbon prices. We use a numerical approach using the LSTM model, which is effectively and accurately able to estimate the parameters μ and σ in a long-term pattern. From the results of accurate parameter estimation, we can see an increase in model performance in simulating carbon price movement predictions. The most accurate carbon price movement prediction results are used to measure the risk of carbon price volatility using the VaR and Conditional Value at Risk (CVaR) methods. The benefits of accurate carbon price movement predictions will also improve VaR and CVaR performance in measuring carbon price movement risks.

Carbon trading research has many challenges in determining risk accurately, so it requires a novel method to get good prediction results. The novelty of this study is the use of geometric Brownian motion modified with LSTM as an estimator parameter to increase the accuracy of measuring carbon price volatility risk. Previously, research has been conducted to predict carbon prices using geometric Brownian motion [9], but research using a hybrid geometric Brownian motion approach with LSTM to predict carbon price fluctuation risk is still limited. Therefore, we contribute to increasing the accuracy of measuring carbon price fluctuation risk.

2. Methodology

Figure 1. Research Process illustrates the research process. The data consists of historical data on carbon price, Brent oil price, gas price, and coal price. Data has gathered from the Yahoo Finance website. The data consists of the daily working days from September 2019 to November 2023. Data preprocessing is done by cleaning, normalizing, and transforming data to prevent bias.

 ISSN 2723-6471

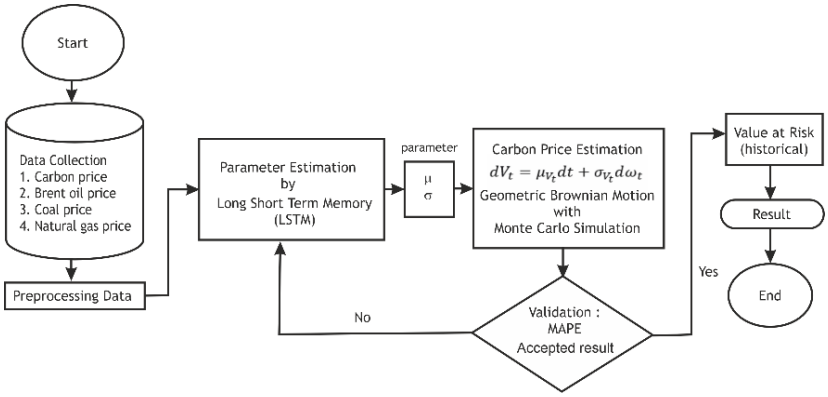

Figure 1. Research Process

One of the SDE-based models is GBM. The parameters of GBM are μ and σ. In estimating GBM parameters, we use LSTM based on the forecasts of carbon prices derived from carbon price variables and energy commodity prices. Then, we input the estimated parameter into the GBM equation and simulated the prediction of a carbon price with Monte Carlo. We used mean absolute percentage error (MAPE) to validate the prediction result and compared the accuracy level of prediction results with GBM without LSTM. The result of the prediction utilizes Value at Risk to assess the risk associated with carbon trading using risk variables related to carbon price volatility.

2.1. Geometric Brownian Motion (GBM) Model with Monte Carlo Simulation

Suppose P_t denotes the carbon return at time t. The GBM model satisfies the following stochastic differential equation (SDE) [17]:

$$dP_t = \mu P_t dt + \sigma P_t d\omega_t \quad (1)$$

dP_t is the change in carbon price, μ_{Pt} is the drift parameter, σ_{Pt} is the volatility parameter and $d\omega_t$ is the increment of the BM. Thus from Eq. (1) $\mu_{P_t}dt$ is the certain component, which controls the trend of the BM trajectory, while $\sigma_{Pt}d\omega_t$ is the uncertain component which controls the random noise effect in the trajectory. Alternatively, the SDE (1) can be expressed in a discretised form as Eq. (2)

$$\frac{\Delta P_t}{P_t} = \mu_{P_t}\Delta t + \sigma Z\sqrt{\Delta t} \quad (2)$$

ΔP_t represents the difference in returns within a brief period t, whereas Z denotes the standard normal random variable. Then the Eq. (2) used to predict carbon price using Monte Carlo Simulation.

Monte Carlo Simulation for GBM is a numerical technique employed to approximate the trajectory of carbon pricing over a period [22]. The subsequent instructions outline the necessary procedures for doing a Monte Carlo simulation for GBM as follows:

We calculate the parameters of the GBM model by setting the carbon price (P), standard deviation (σ), average carbon price (μ), and time (T). Next, we determine the number of iterations (n) for the Monte Carlo simulation and set the initial stock price (P_0) to a predetermined value. Next, we use the GBM formula to model the change in carbon price in each iteration with the formulation in Eq. (3).

$$P_t = P_{t-1} \times e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t Z_t}} \quad (3)$$

In this context, P_t represents the carbon price at a certain time t, P_{t-1} refers to the carbon price at the previous time, μ denotes the average carbon price, σ represents the standard deviation, Δt represents the time interval, and Z_t is a random variable. Next, we run this simulation repeatedly according to the number of iterations set and save the carbon price from each iteration for further analysis. The visualization results of the carbon price distribution will then be analyzed for changes over time, and the simulation results will be improved by performing repeated iterations until the smallest MAPE value is reached.

 ISSN 2723-6471

The stochastic differential equation model was enhanced by iterating the carbon price across 10, 100, 1000, 10000, 100000, and 1000,000 paths. The trajectories are optimized by comparing MAPE data to achieve the most accurate and best results. A Monte Carlo simulation is used to determine the optimal path by considering small error values.

2.2. Long Short-Term Memory

LSTM has the benefit of long-term memory more effectively because it allows more parameters to be learned [5]. LSTM can capture the characteristics of complex nonlinear time series relationships from various data [19]. LSTM is also consistent with its capacity to extract complex nonlinear correlations from financial data. LSTM implies that ML techniques can greatly improve the accuracy of predictions in various domains [20].

This study utilizes the LSTM approach to estimate the parameters μ and σ of carbon prices. LSTM is a Deep Learning network designed for analyzing sequential data. LSTM networks excel at retaining both short-term and long-term information within the network [5]. LSTM is composed of cells with input, output, and memory gates. These three gates control the information flow. Each cell retains the specified value for a variable period with this capability. LSTM cells aggregate to create brain tissue layers. Figure 2. LSTM flowdepicts the fundamental components of LSTM, including σg (sigmoid function), tanh (hyperbolic tangent function), ⊗ (multiplication), and addition.

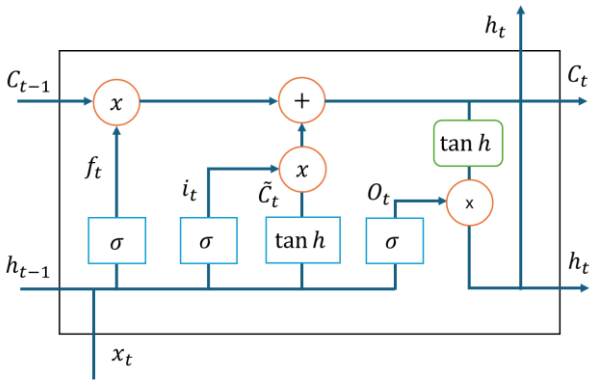

Figure 2. LSTM flow

The forget gate is the initial gate identified, determining the degree to which information from the previous cell's concealed state should be disregarded. This is achieved using a sigmoid function, as shown in Eq. (4). Subsequently, the input gate is identified, which is accountable for controlling the amount of fresh information to be stored in the present cell state. The input gate utilizes a sigmoid Eq. (5) and introduces a new tanh function, as shown in Eq. (6) and Eq. (7). The output gate is shown in Eq. (8), and it determines the new value and the hidden state in Eq. (9) that will be the output of the current cell. This is accomplished by using the sigmoid function. The equations within an LSTM cell can be expressed formally as follows:

$$f_t = \sigma\left(W_f.[h_{t-1}, x_t] + b_i\right) \quad (4)$$

$$i_t = \sigma\left(W_f.[h_{t-1}, x_t] + b_i\right) \quad (5)$$

$$\tilde{c_t} = tanh\ (W_c.[h_{t-1}, x_t] + b_c) \quad (6)$$

$$c_t = f_t * c_{t-1} + i_t * \tilde{c}_t \quad (7)$$

$$o_t = \sigma\ (W_0.[h_{t-1}, x_t] + b_o) \quad (8)$$

$$h_t = o_c \otimes tanh(c_t) \quad (9)$$

Where ⊗ represent the dot product, h_{t-1} denotes previous output, C_{t-1}, C_t, and $\tilde{C}_t$ are previous memory state, current state and intermediate state. And then i_t, f_t, o_t are the added degree value, retained degree value and current degree value state, h_t is output data, and x_t denotes the input time series. These three gates determine what to store, update, or forget from the internal memory, generating the hidden representation at each step. The tanh activation function ensures that the output of the neurons is in the range [-1,1] to maintain stability during information propagation. The model is compiled with the Adam optimizer, which adaptively regulates the learning, with the loss function being MSE to minimize the difference between predictions and targets and using the accuracy value as a metric to evaluate the model's performance.

 ISSN 2723-6471

2.3. Deep Multilayer Perceptron

Deep multi-layer perceptron (DMLP) is the one of the most network model that can be used to estimate carbon price. Previous studies showed that the model has produced high prediction accuracy and a model-fit effect [23]. Several types of single models (ARIMA, ANN, LSSVM), hybrids, and variants exist in previous carbon price research in phase II [24]. Regarding its predictive ability, the MLP is better than all single models and some hybrid models [23]. Based on existing research, we are interested in comparing DMLP with LSTM in making predictions.

DMLP is a development of conventional MLP. Its neuron architecture consists of an input, hidden, and output layer. The difference with MLP is that DMLP has more neuron layers than regular MLP. The DMLP has input components in each neuron, including (x), weights (w), biases (b), and output (y) [23].

The output of neurons in a neural network is illustrated in Eq. (10). Each neuron with a nonlinear activation function generates an output by assembling weighted inputs from neurons in the previous layer. The activation functions often used in DMLP include sigmoid, linear, and unit steps [23].

$$y_i = \sigma(\sum_i W_i x_i + b_i) \tag{10}$$

The activation function can take various forms, as listed in Table 1. Model accuracy testing [17].

Table 1. Model accuracy testing [17]

Activation Function	Equation
Unit step	$f(z) = \begin{cases} 1 z \geq 0 \\ 0 z < 0 \end{cases}$
Linear	$f(z) = z$
Logistik (sigmoid)	$f(x) = \frac{1}{1+e^{-x}}$

DMLP can learn functions that can be linearly separated, as stated in Eq. (10). Figure 3. Input patterns, from left to right: a. linear separable, b. nonlinear separable [3]shows an example of a linear function that divides the data into two classes.

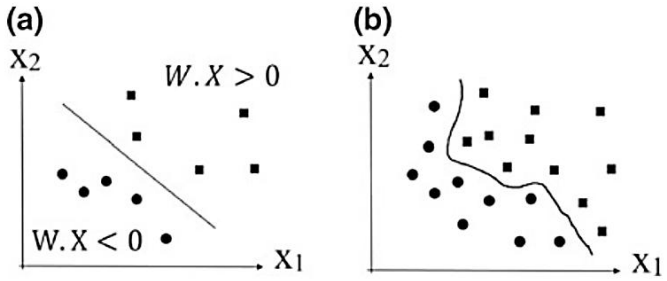

Figure 3. Input patterns, from left to right: a. linear separable, b. nonlinear separable [3]

With DMLP, more efficient classification and faster regression performance can be achieved compared to models with fewer layers. The DMLP learning process is done through backpropagation, where errors in the output layer neurons are also traced back to the neurons in the previous layer. In DMLP, the stochastic gradient descent (SGD) method is used for learning optimization (updating weights between layers). Figure 4. Deep Multilayer Perceptron diagram illustrates the scheme of the DMLP model, layers, neurons, and weights.

 ISSN 2723-6471

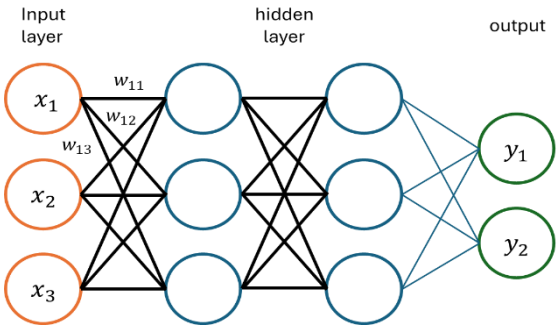

Figure 4. Deep Multilayer Perceptron diagram

DMLP involves input, hidden, and output layers [23]. The input layer receives input from data consisting of carbon and energy prices. The input node receives valuable information from the predictor variables. Hidden layers are responsible for extracting input data patterns in neural networks. In the hidden layer, each neuron is interconnected, allowing the model to learn nonlinear relationships between input and output. The output layer receives information as signals from the hidden layer. In this DMLP model, there are several input variables and one output. Each node in the output layer calculates the output value based on the weight and bias values obtained from the previous layer. The output measures the model's error through the loss function and compares the predicted and target values. The output results are used to update the weights and biases to minimize the prediction error.

2.4. Performance Test

In this study, we chose performance testing from simulation using MAPE, which is defined in Eq. (11).

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\frac{|\hat{y}_i - y_i|}{y_i} * 100 \quad (11)$$

where $\hat{y}_i$ is the predictedted carbon price, y_i is, the actual carbon price, and n is the amount of data. Table 2. Model accuracy testing [17]shows the scale of several accuracy values when simulating using MAPE. The lower the MAPE value, the more accurate it is when estimating future predictions [16], [25].

Table 2. Model accuracy testing [17]

MAPE	Judgement of forecast accuracy
$\leq 10\%$	Exceptionally precise
11% *to* 20%	Optimistic forecast
21% *to* 50%	A Reasonable forecast
$> 50\%$	A forecast that is not accurate

2.5. Carbon Trading Risk Measurement

The dynamic movement of carbon prices causes a relatively high level of volatility, so there is still a suspicion that the data is not entirely customarily distributed, so risk measurement uses the historical VaR and CvaR methods. The historical simulation method is one method that can be used directly to calculate VaR because it does not require assumptions about normality [26]. VaR is usually expressed in a certain period. This value is associated with a level of confidence, which is often expressed as a percentage (for example, 95% or 99%). A VaR of 95% means a 5% chance that the loss may exceed the estimated value, while a VaR of 99% means a 1% chance that the loss may exceed the estimated value.

The following are the steps in risk measurement [27]. We simulate *return values* by randomly generating n single asset *returns* with the parameters obtained from step (1) so that an empirical distribution of simulated *returns is formed.* We look for the maximum loss estimate at the confidence level (1 – α), namely as the α-quantile value of the empirical distribution of *returns* obtained in step (2), denoted by R*. We calculate the VaR value at confidence level (1 - α) in a time of t days, the historical VaR represent to Eq. (12):

$$\text{VaR}_{(1-\alpha)}(t) = V_0 R^* \sqrt{t} \quad (12)$$

with V_0 = initial investment funds. R* = α-th quantile value of the *return distribution.*

 ISSN 2723-6471

CVaR calculate losses that exceed the VaR level [13]. The CVaR value can be expressed in Eq. (13) as follows

$$\text{CVaR} = \mathbb{E}[\text{L}|\text{L} > \text{VaR}_\alpha] \quad (13)$$

where L is price losses, VaR_α is VaR in confidence α

After that we repeat steps (2) to step (4) *m times* so that it reflects various possible *VaR values* for a single asset, namely $VaR_1, VaR_2, \ldots, VaR_m$. Final step we calculate the average results from step (5) to stabilize the value because the *VaR value* produced by each simulation is different.

3. Results and Discussion

In this section, we explain the research results and discussion completely by dividing it into several subsections, including data source, preprocessing data, GBM parameter estimation using LSTM, Monte Carlo simulation, and risk measurement.

3.1. Data Source

The data used is sourced from Yahoo Finance in accordance with research parameters including daily oil price data (Crude Oil/CL-F), daily coal price data (MTF-F), daily natural gas price data (Natural Gas/NG-F), and daily carbon price (GRN) data. Yahoo Finance site provides financial news, company financial information, and market-related information, including potential issues that are factors in price increases or decreases, including carbon prices. Yahoo Finance is one of the right choices for research, especially in the financial field, because the data on Yahoo Finance is needed for research and for people who invest or trade stocks [28]. We use data from 2019 to 2023 in this research, represented in Table 3. Head of Dataset

Table 3. Head of Dataset

Date	Carbon Price	Coal Price	Oil Price	Gas Price
9/18/2019	10.13	60.95	63.07	2.64
9/19/2019	10.13	60.95	63.47	2.54
9/20/2019	10.615	61.25	64.05	2.53
…	…	…	…	…
11/7/2023	30.101999	116.3	81.02	3.14
11/8/2023	30.040001	115.68	80.42	3.13

We use several variable data in this study, including carbon price data and energy commodity price data (oil, gas, and coal). Figure 5. GRN Carbon and Energy Commodity Price. shows a visualization of the data movement. The x-axis represents time in days, and the y-axis represents carbon prices and energy commodity prices in USD. Carbon prices are marked in blue with the most stable movements. Coal prices are marked red, Brent oil prices are marked in light green, and gas prices are marked in blue. Energy commodity prices have very dynamic price movements.

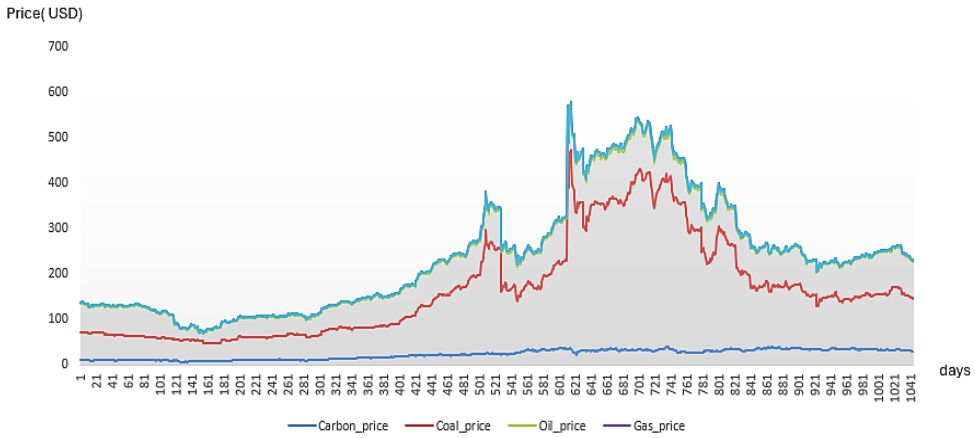

Figure 5. GRN Carbon and Energy Commodity Price

 ISSN 2723-6471

3.2. Preprocessing Data

In data preprocessing, outliers are caused by spikes or drops in energy prices; outliers are identified by visualizing carbon data and energy commodities. So, in overcoming outliers, we use data transformation namely all data sets are processed with normalization represent in Eq. (14) as follows:

$$X = \frac{x_i - x_{min}}{x_{max} - x_{min}} \quad (14)$$

Where x_i is the value of the i-th sample, x_{min} is the minimum value of the sample, and x_{max} is the maximum value of the sample. Based on the formula above, the data of all prediction indices ranges from 0 to 1. After normalization, we set carbon prices and energy commodities as input variables handled by LSTM to estimate parameters [5].

3.3. GBM Parameter Estimation using LSTM

The LSTM process of estimating parameters begins with inputting data, which consists of historical carbon data and energy commodity prices. Before the input is entered into the LSTM, we must preprocess the data with min-max data transformation with a scale of [0,1]. Next, we enter the transformed data into the LSTM model. Then, calculate the return value of the carbon price represent in the Eq. (15) as follows:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (15)$$

P_t is the current carbon price, and P_{t-1} is the previous carbon price

The LSTM model architecture we built has 12 neurons using the tanh activation function. Furthermore, the data is trained with 80% training data and 20% test data, with a batch size of 64 samples and a buffer size 128. Training is carried out for 100 epochs, each consisting of 100 steps. The model processes the batch at each step, calculates predictions, and updates weights through backpropagation. After all epochs have been passed, the model is ready to predict new data based on the temporal patterns learned.

Next, evaluate the model using a loss function; the loss function used is MSE using the Eq (16):

$$MSE = \frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{n} \quad (16)$$

$\hat{y}_i$ predicted value and y_i actual value

Furthermore, the means parameter value is obtained from the average value of the carbon price log return prediction, and the volatility is obtained from the standard deviation value of the carbon price log return prediction.

The parameters estimated are the parameters μ_{vt} which are the drift coefficient (average return over several time periods), the drift coefficient on the carbon price parameter is obtained using the Eq. (17).

$$\mu = \frac{1}{n}\sum_{i=1}^{n} R(t_i) \quad (17)$$

$R(t_i)$ is the return value of the predicted carbon price obtained from LSTM, and σ_{vt}which is the diffusion coefficient (standard deviation *of return*). The coefficient of standard deviation is obtained in the Eq. (18).

$$s^2 = \frac{1}{n-1}\sum_{t-1}^{n}(R(t_i) - \bar{R})^2 \quad (18)$$

Next, the estimated parameter value obtained, the mu (μ) are entered into the Geometric Brownian Motion Eq. (19)

$$dP_t = \mu P_t dt + \sigma P_t d\omega_t \quad (19)$$

From equation (14), a Monte Carlo simulation is then carried out to find the most optimal path; the simulation uses iterations of 10, 100, 1000, 10000, 100000, and 1000000. From each iteration, the most accurate result will be selected.

3.4. Monte Carlo Simulation

The subsequent section presents the outcomes of carbon price simulations utilizing Geometric Brownian Motion, conducted through iterations of 10, 100, 1000, 10000, 100000, and 1000000. Subsequently, a comparison is made between the actual results and the simulation outcomes, followed by a comparison with the average simulation value. Figure 6. Results of Carbon Price Model Simulation. represents the prediction of the GRN carbon price movement. In

ISSN 2723-6471

the image caption, the horizontal coordinates represent daily time series, and the vertical coordinates represent the carbon price (GRN). The explanation in Figure 6 are as follows: Prediction 1 describes the results for 10 iterations, prediction 2 describes the results for 100 iterations, prediction 3 describes the results for 1000 iterations, prediction 4 describes the results for 10000 iterations, prediction 5 describes the results for 100000 iterations, and prediction 6 describes the results for one million iterations.

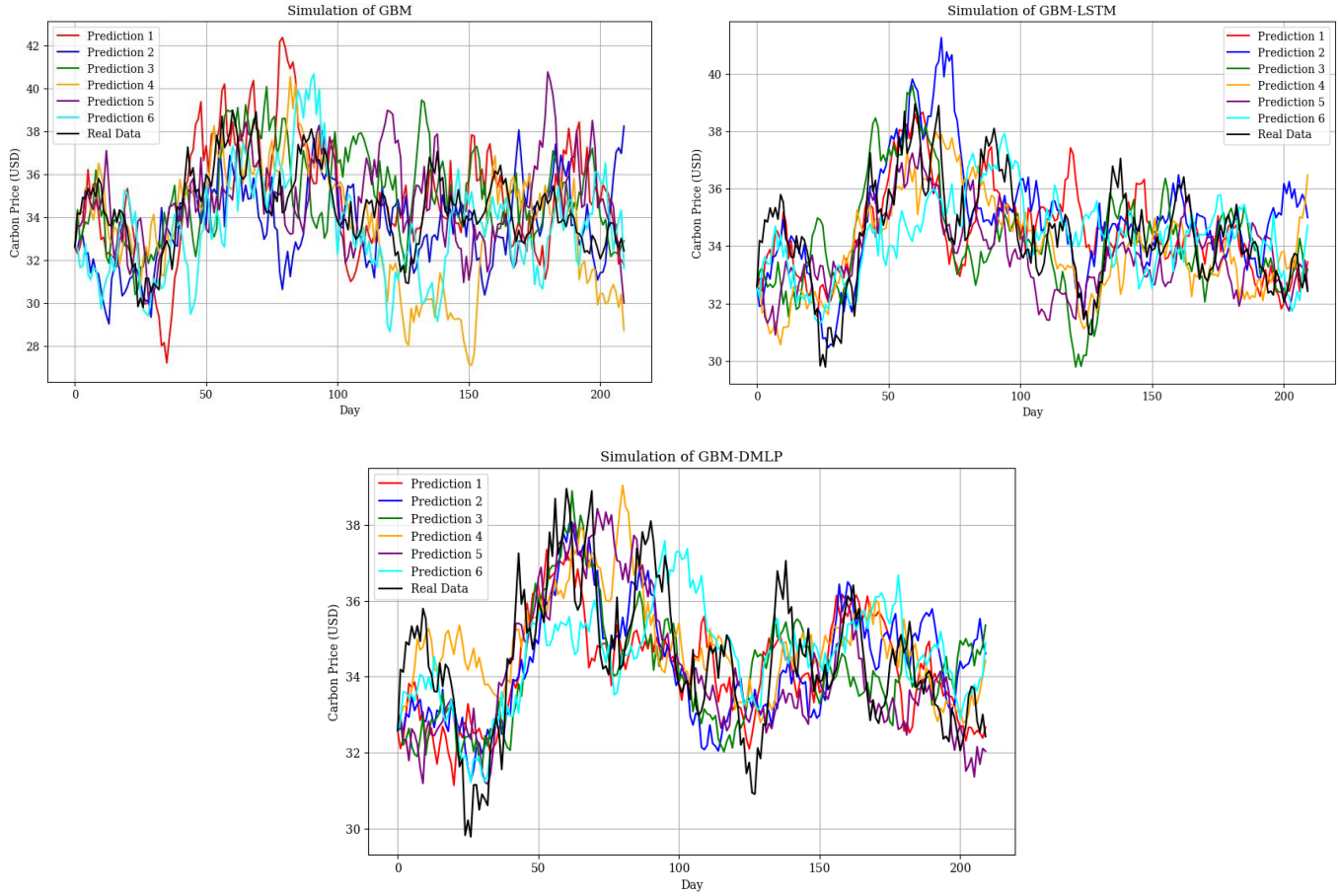

Figure 6. Results of Carbon Price Model Simulation

Simulation results using the Monte Carlo method are used to evaluate model performance. In this context, we compare the GBM model with the GBM-LSTM and GBM-DMLP model, as shown in Table 4. Carbon Price Prediction Results and MAPE Values

Table 4. Carbon Price Prediction Results and MAPE Values

No	Number of Iterations	GBM (%)	Time (sec)	GBM-LSTM (%)	Time (sec)	GBM-DMLP	Time(sec)
1	10	8.35	1.021	5.42	55.439	6.65	9.221
2	100	7.18	1.327	4.63	58.874	5.32	7.270
3	1000	6.72	4.243	4.29	65.632	4.60	10.124
4	10000	5.63	27.453	4.09	92.340	4.31	42.335
5	100000	4.96	300.321	3.66	363.203	4.11	350.465
6	1000000	4.77	3403.352	3.14	3482.013	3.81	3443.674

 ISSN 2723-6471

From the simulation results in Table 4. Carbon Price Prediction Results and MAPE Values, the more trajectories there are, the better the model performance will be. This is indicated by a decrease in the error value on each trajectory. For the same input variables, namely carbon price and energy commodity price, the MAPE value in the GBM-LSTM model is the smallest compared to the DMLP-LSTM and GBM models. This means that the GBM-LSTM prediction is better than the DMLP-LSTM and regular LSTM because LSTM is better able to recognize nonlinear carbon price time series. Then, LSTM can memorize crucial patterns in the long term, resulting in higher accuracy.

Regarding the iteration time in making predictions, the GBM-LSTM hybrid method has the longest time compared to the GBM and GBM-DMLP methods because the LSTM architecture has the most complex components, especially in remembering long-term memory and forgetting irrelevant information in memory. Thus, although the GBM-LSTM model requires a longer time, this extended time is still comparable to the accuracy value of the other models.

Monte Carlo simulations also depend heavily on the magnitude of the path value. The greater the path value, the more simulations are carried out. From the number of simulations, the slightest error value is obtained. The relatively long computation time is still comparable to the accuracy value obtained at high iterations (where the best performance is obtained), and the difference in computation time of the three algorithms is not too significant. The long computation time is used for the training process, while the training process is not carried out continuously. When applied, the testing process time for the three algorithms does not take long.

The next step is to measure the risk of loss due to carbon price fluctuations. Dynamic carbon price movements cause relatively high volatility, so there is still a suspicion that the data is only partially distributed risk measurement using the historical VaR and CVaR methods. The Historical Simulation method is one method that can be used directly to calculate VaR because this method does not require normality assumptions [26]. Then, we present the results of the risk measurement in Table 5. The result of risk measurement.

Table 5. The result of risk measurement

No.	Methods	Historical VaR (%)		CVaR (%)	
		$\alpha = 0.05$	$\alpha = 0.01$	α =0.05	α =0.01
1.	GBM	3.99	6.83	5.44	6.22
2.	GBM-LSTM	2.34	3.24	2.93	3.77
3.	GBM-DMLP	1.80	2.47	2.23	2.90

Based on Table 5. The result of risk measurement, for the historical simulation method, it can be interpreted that, with an initial investment of USD 100 for carbon prices at a 95% confidence level, the highest risk of loss is the GBM method with the amount of loss that investors may bear not exceeding USD 3.99. Still, in the historical simulation with a 99% confidence level, the value of the loss that investors may bear will not exceed USD 6.83. The lowest predicted value of VaR or CVaR is in the GBM-DMLP method, while the highest risk value is in the GBM method. The risk of loss at $\alpha = 0.01$ shows a tendency for a higher value than the risk of loss with $\alpha = 0.05$, indicating that the higher the confidence level, the higher the loss value. The comparison between historical VaR and CVaR shows that the value of CVaR is higher than that because CVaR calculates the average value of extreme losses. From the results of the risk measurement after obtaining the VaR and CVaR values, we tested the validity or accuracy of VaR using the backtesting method. Table 6 presents the results of backtesting the VaR and CVaR risk measurements.

Table 6. The result of backtesting VaR

No.	Methods	Number of Failure (%)			
		Historical VaR $\alpha = 0.05$	Historical VaR α =0.01	CVaR α=0.05	CVaR α=0.01
1.	GBM	4.07	1.63	1.63	0.81
2.	GBM-LSTM	4.79	1.20	1.20	0.03
3.	GBM-DMLP	6.59	1.20	1.80	1.20

Based on Table 6. The result of backtesting VaR, historical risk prediction with α=0.05, the lowest failure rate is obtained in the GBM model, while the highest is in GBM-DMLP. Furthermore, in historical risk prediction with $\alpha =$

 ISSN 2723-6471

0.01, the lowest failure rate is in the GBM-LSTM method, while the highest is in the GBM method. Furthermore, in CVaR measurement, the highest failure rate is in the GBM-DMLP method, while the lowest is in the GBM-LSTM method. The results of risk measurement with CVaR show better results than historical VaR, which can be proven in Table 4, where the failure rate in CVaR shows a smaller value than the failure rate in historical VaR. The difference in VaR and CVaR measurements proves that the GBM-LSTM model is reliable in estimating parameters. The proper parameters will affect the results of carbon price prediction and volatility risk prediction. In this risk forecast, the time horizon used is short-term because it uses daily carbon data and energy commodity data. Its function is to determine the risk characteristics of the carbon market, especially in Europe, and the magnitude of the risk caused by carbon price fluctuations.

4. Conclusion

This paper proposes a hybrid GBM-LSTM model to predict carbon price volatility risk. The main processes include data preprocessing, parameter estimation, carbon price estimation, and risk measurement. Simulation results indicate that the GBM-LSTM model is better than the DMLP-GBM and regular GBM. GBM-LSTM significantly reduces the error at each iteration. The decrease in error also establishes that combining GBM and LSTM improves the model's ability to recognize more complex carbon price data patterns, especially nonlinear ones. The limitation of relatively long computation time is still comparable to the accuracy value obtained at high iterations (where the best performance is obtained), and the difference in computation time of the three algorithms is not too significant. The difference in VaR measurements between predicted and actual data proves that the GBM-LSTM model is reliable in estimating parameters and, with the correct parameters, will affect the results of carbon price prediction and risk prediction of carbon price volatility. The research of the hybrid LSTM method with GBM is a good start in exploring the characteristics of carbon price fluctuations at a particular time. Integration with a more sophisticated neural network architecture will increase the accuracy and function value of optimizing carbon trading risk. In the following research, it can be developed further by developing a more optimal deep learning model for measuring risk.

5. Declarations

5.1. Author Contributions

Conceptualization: Y.A.P., I.M., M.I.I., E.R.P., and M.I.; Methodology: I.M., E.R.P.; Software: Y.A.P.; Validation: Y.A.P., I.M., and M.I.; Formal Analysis: Y.A.P., I.M., and M.I.; Investigation: Y.A.P.; Resources: I.M.; Data Curation: I.M.; Writing Original Draft Preparation: Y.A.P., I.M., M.I.I., E.R.P., and M.I.; Writing Review and Editing: I.M., Y.A.P., and M.I.; Visualization: Y.A.P. All authors have read and agreed to the published version of the manuscript.

5.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

5.3. Funding

We thank The Ministry of Education, Culture, Research, and Technology for funding this research through the Postgraduate Research Scheme, with contract number 1707/PKS/ITS/2024.

5.4. Institutional Review Board Statement

Not applicable.

5.5. Informed Consent Statement

Not applicable.

5.6. Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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