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Hybrid MINLP-FNS Framework for Solving Large-Scale LIRP in E-Retail Logistics

Kristian Telaumbanua[1,], Syahril Efendi[2,*], Poltak Sihombing[3,], Maya Silvi Lydia[4,]

[1]Doctoral Student, Faculty of Computer Science, Information Technology Engineering Program, Universitas Sumatera Utara, North Sumatera, Indonesia

[2,4,3]Faculty of Computer Science, Information Technology Engineering Program, Universitas Sumatera Utara, North Sumatera, Indonesia

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Abstract

This research proposes a hybrid optimization framework to address the large-scale, multi-echelon Location-Inventory-Routing Problem (LIRP) in e-retail logistics. The proposed method combines a Mixed-Integer Nonlinear Programming (MINLP) model with a tailored Feasible Neighborhood Search (FNS) algorithm to solve complex decision-making problems involving facility location, inventory control, and vehicle routing simultaneously. Distinct from conventional models, the framework is integrated with a Business Intelligence (BI) environment to enable real-time decision support and dynamic data processing. Experimental evaluations were conducted using realistic logistics scenarios involving four distribution echelons. The results show that the hybrid MINLP-FNS approach achieves a total logistics cost reduction of 13.6% and a runtime improvement of 48% compared to the baseline MINLP-only model. It also significantly outperforms traditional GRG-based methods in scalability and computational stability across large datasets. These findings demonstrate that the proposed hybrid framework offers a more effective and scalable solution for complex logistics optimization, while its BI integration ensures practical applicability in real-world operations. This study contributes a novel data-driven framework that advances current research in intelligent supply chain and optimization systems.

Keywords: Hybrid Optimization, MINLP, FNS, LIRP, E-Retail Logistics, Business Intelligence, Supply Chain Optimization

1. Introduction

The exponential growth of e-commerce has fundamentally transformed the logistics landscape, particularly in e-retail systems, where service responsiveness and cost efficiency are critical to competitiveness [1], [2]. One of the central challenges in this domain is the Location-Inventory-Routing Problem (LIRP), which simultaneously determines the optimal locations for distribution centers, inventory allocation, and vehicle routing across multi-echelon supply chains [3], [4], [5]. The LIRP has been widely recognized as an NP-hard problem due to its combinatorial nature and integrated decision variables [6], [7]. To address LIRP, various optimization approaches have been proposed, ranging from exact methods such as Mixed-Integer Programming (MIP) and Mixed-Integer Nonlinear Programming (MINLP) [8], [9], to heuristics and metaheuristics including Genetic Algorithms (GA) [10], [11], Particle Swarm Optimization (PSO) [12], [13], and Ant Colony Optimization (ACO) [14].

While exact models offer high precision, their scalability is limited in large-scale systems due to computational complexity. On the other hand, metaheuristics provide faster solutions but may suffer from local optima and lack of convergence guarantees [15], [16]. Despite these developments, most existing LIRP studies focus on static and small-scale scenarios, often neglecting the dynamics and scale of real-world e-retail supply chains [17], [18]. Moreover, these methods typically operate in isolation from modern data environments, lacking integration with Business Intelligence (BI) systems that support adaptive, data-driven logistics decision-making [19], [20]. This disconnect reduces their practical applicability in contexts that require real-time responsiveness and multi-source data processing. In particular, conventional MINLP models, although powerful, often become computationally infeasible

*Corresponding author: Syahril Efendi (syahril1@usu.ac.id)
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when applied to multi-echelon networks with large numbers of customers and distribution points [21], [22]. Furthermore, standard heuristics fail to effectively exploit problem structure in MINLP-based LIRP, leading to suboptimal solutions or excessive computation time [23], [24]. To overcome these limitations, this study proposes a hybrid optimization framework that integrates a continuous Feasible Neighborhood Search (FNS) heuristic into an MINLP model, specifically tailored for large-scale LIRP in four-echelon e-retail logistics systems. In addition, the framework is embedded within a Business Intelligence platform, enabling seamless integration with operational data and enhancing the responsiveness of supply chain decisions [25]. The FNS used in this study was tailored with three structural modifications: a MINLP-guided neighborhood generator, an adaptive acceptance probability to escape local minima, and a multi-layer decision operator specifically designed for the LIRP's joint location–inventory–routing structure. These extensions distinguish the proposed FNS from conventional FNS implementations. The hybrid approach is designed to balance between solution quality and computational efficiency by leveraging heuristic exploration and mathematical rigor in a unified framework.

For instance, large-scale MINLP instances in refinery scheduling [21] require up to several hours for problems with 50–100 binary variables, and energy MINLP models reported by [22] become numerically unstable beyond 200 constraints. The objective of this research is to develop, implement, and evaluate a scalable hybrid optimization model that minimizes total logistics costs while maintaining service level constraints across complex distribution networks. The proposed solution is benchmarked against traditional methods and is validated using realistic e-retail logistics data, demonstrating its practical feasibility and superior performance in large-scale environments.

2. Literature Review

The LIRP has gained significant attention in logistics and supply chain optimization due to its ability to simultaneously address three interconnected decision-making components: facility location, inventory control, and vehicle routing [26]. Early LIRP studies typically focused on simplified two-echelon structures, often using MIP or MILP to solve small-scale problems [27]. However, these approaches face substantial scalability limitations when applied to complex multi-echelon networks, such as those found in e-retail logistics. In response, recent research has adopted MINLP models to better capture the nonlinear cost structures and interaction effects in large-scale distribution systems [28], [29]. MINLP enables more accurate modeling of transportation costs, inventory holding costs, and service-level constraints. Nevertheless, solving MINLP models remains computationally challenging, particularly for high-dimensional LIRP instances [30], [31].

To overcome this, a variety of heuristic and metaheuristic algorithms have been introduced, including GA, PSO, Tabu Search (TS), and ACO [32], [33]. These methods offer faster computational performance but often sacrifice optimality, especially in highly constrained and interdependent decision environments. Moreover, most heuristic approaches assume a static environment and lack integration with real-time data systems. Another major limitation in prior studies is the absence of BI integration. Despite the increasing adoption of BI tools in logistics operations, only a few optimization models leverage BI platforms to enable adaptive, data-driven decision-making [34]. This presents a significant research gap, particularly in the context of e-retail logistics where responsiveness and agility are critical.

Several recent studies have adopted hybrid optimization approaches combining mathematical programming with heuristics or metaheuristics to balance solution quality and computational efficiency. For example, Goodarzian et al. propose a hybrid MILP-based metaheuristic for optimizing sustainable agricultural supply chains, combining exact MILP with metaheuristics to handle real-world complexities [35]. In another instance, Liu et al. develop a MINLP model for a logistics center location-inventory-routing problem and solve it using genetic algorithms to address large-scale network instances [5]. Furthermore, the propose a hybrid method (HMCGP-UFGA) for a perishable food supply network's integrated location-inventory-routing problem under disruptions, combining goal programming heuristics and genetic algorithm components [32]. Most LIRP studies including [6], [32], and [5] operate purely in optimization environments without BI-enabled data pipelines. Only limited attempts (e.g., [34]) explore BI-oriented decision dashboards, but not integrated optimization-BI frameworks.

Despite these advances, such approaches often remain constrained in several respects: (i) limited to two- or three-level networks (not full multi-echelon systems), (ii) weak coupling between exact and heuristic modules, and (iii) lack of real-time decision support embedded in BI environments. In the context of e-retail logistics, these shortcomings impede practical applicability under dynamic, large-scale settings. To fill this gap, this dissertation proposes an integrated MINLP–FNS framework. It combines rigorous modeling via MINLP with a tailored FNS algorithm to efficiently navigate the solution space. Critically, the framework is embedded within a BI environment to support real-time, data-driven logistics decisions. Consequently, this approach addresses scalability, algorithmic integration, and practical deployment issues in dynamic multi-echelon e-retail logistics. This literature synthesis positions the present work as an advancement over prior studies by: (i) scaling to multi-echelon, multi-period networks; (ii) tightly integrating exact and heuristic methods within a unified framework; and (iii) enabling interactive, BI-facilitated decision making in logistical operations.

3. Methodology

3.1. Dataset

This study uses a simulated dataset that reflects the hierarchical structure of an e-retail logistics system consisting of four echelons: central warehouses, regional warehouses, distribution agents, and end customers. This structure was developed based on a real-world configuration from a large Indonesian e-retail company to ensure the relevance and applicability of the model to actual conditions. The network comprises 2 central warehouses, 5 regional warehouses, 20 distribution agents, and 60 end customers, amounting to 87 nodes in total. Goods flow through the system in a top-down manner: from central warehouses to regional warehouses, then to distribution agents, and finally to customers. Each customer is exclusively served by a single distribution agent. The distance between nodes is calculated using the Euclidean distance, based on assigned coordinates. Customer demands are deterministic, and each node has predefined service level targets. The dataset includes logistics parameters such as transportation costs, warehouse capacities, vehicle capacities, and shortage penalties, which serve as input values for the proposed model. All parameter values were validated through consultations with logistics practitioners familiar with the operational landscape of e-commerce logistics in Indonesia.

3.2. Research Framework

The research framework was systematically designed to develop and validate a hybrid optimization approach for solving the Location-Inventory-Routing Problem (LIRP) in e-retail logistics networks. The framework integrates problem formulation, optimization modeling, heuristic algorithm development, simulation analysis, and decision-support implementation to ensure that the proposed approach is both computationally efficient and practically applicable in real-world logistics operations. The overall framework consists of five main stages, namely problem identification, model design, algorithm development, simulation and validation, and decision-support integration. The first stage focuses on identifying the operational challenges encountered in large-scale and multi-echelon e-retail logistics systems, particularly those related to the simultaneous optimization of facility location, inventory management, and vehicle routing. Based on these challenges, the second stage formulates the problem into a Mixed-Integer Nonlinear Programming (MINLP) model capable of representing the interdependencies among logistics components and operational constraints.

Subsequently, the third stage develops a Feasible Neighborhood Search (FNS) heuristic algorithm to address the high computational complexity commonly associated with large-scale MINLP optimization problems. To evaluate the effectiveness of the proposed approach, the fourth stage performs simulation and validation using a generated logistics dataset. The evaluation is conducted based on several performance indicators, including total logistics cost, order fulfillment rate, and transportation distance. Finally, the fifth stage integrates the proposed hybrid optimization model into a business intelligence environment to support strategic and operational decision-making processes in e-retail logistics management. Figure 1 illustrates the overall structure of the proposed research framework. The figure demonstrates the sequential relationship between each research stage, beginning with problem identification and continuing through model formulation, heuristic optimization, simulation analysis, and decision-support integration.

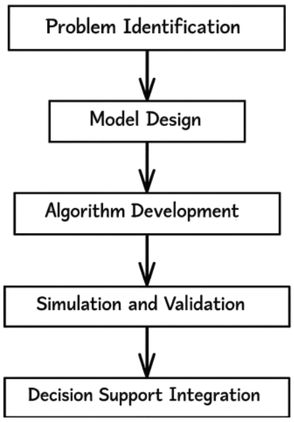

Figure 1. Research Framework

The framework shown in figure 1 provides a comprehensive representation of the proposed research methodology and highlights the systematic flow of the study. In addition, the framework ensures that the developed optimization model not only achieves computational efficiency but also supports practical implementation in intelligent logistics decision-support systems.

3.3. Algorithm Development

This section presents the development of an integrated algorithm to solve the four-echelon LIRP in an e-retail logistics system. The problem formulation is based on a complex supply chain model that incorporates uncertain customer demand, multi-product distribution, vehicle routing, facility location, and inventory control all represented in the distribution cost flow shown in figure 2.

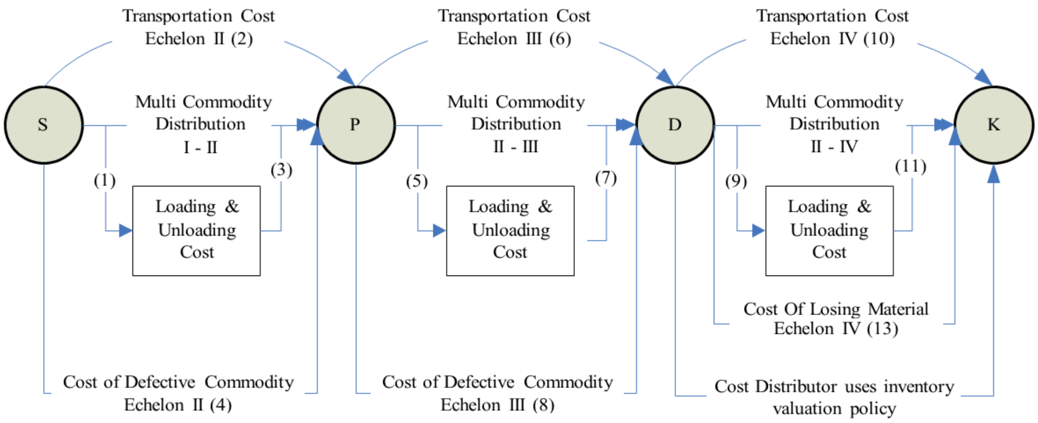

Figure 2. Total Distribution Cost Flow in Four-Echelon E-Retail Logistics

Figure 2 illustrates the cost structure and flow of goods in a four-echelon e-retail logistics network, consisting of suppliers (S), producers (P), distributors (D), and customers (K). The model captures transportation costs, multi-commodity distribution flows, and loading-unloading activities at each echelon. It also accounts for defective commodity costs at the supplier and producer levels, as well as the cost of material loss during the final delivery stage. A continuous (r, Q) inventory policy is applied at the distributor level to manage inventory dynamics. All these components are integrated into the MINLP formulation to comprehensively minimize total logistics costs.

3.3.1 Formulation of the Optimization Model

To formally represent the four-echelon LIRP in the e-retail logistics system, this study develops a MINLP model. The formulation is designed to minimize the total logistics cost while fulfilling a series of practical constraints that reflect real-world operational conditions in supply, transportation, inventory management, and product distribution. The objective function integrates several cost components to reflect the complexity of the logistics network. These include procurement costs incurred when sourcing raw materials from suppliers, both fixed and variable production costs at the manufacturing level, and inventory holding costs for raw materials and finished goods stored throughout the supply chain. Additionally, transportation costs are considered for each stage of the distribution process, spanning from suppliers to producers, then to distribution centers, and finally to customers. The model also accounts for the operational expenses of distribution centers, penalties for unmet customer demand (shortage costs), and vehicle-related expenses such as acquisition and fuel consumption, which are influenced by travel distance and vehicle efficiency. Collectively, these components are formulated into a single nonlinear objective function, which serves as the basis for optimizing the total cost of the entire logistics system under uncertainty and capacity limitations. The objective function is expressed as:

$$\begin{gathered} Min\ \sum_j Fi\ (\min\{1, \sum_m x_{mi}\}) + \sum_j \sum_p AP_{pj}.y_j \cdot \left(\frac{\sum_k \mu_{pk}}{\sum_k \sum_j \sum_v Q_{pvkj}}\right) + \sum_i \sum_m \sum_k C\,MP_{mi}.\mu_{pk} \cdot CO_{mip} \cdot X_{mi} + \\ \sum_j FWS_j \cdot y_j + \sum_i \sum_k \sum_p \sum_v Q_{pvkj}.T_{pj} + \sum_i \sum_p \sum_m A\,M_{mi} \cdot \left(\frac{\sum_k \mu_{pk} \cdot CO_{mip}}{\sum_k \sum_j \sum_v Q_{pvkj} \cdot CO_{mip}}\right) \cdot X_{mi} + \\ \sum_m \left(\frac{\sum_p \sum_k \sum_j \sum_i \sum_v PQ_{pvkj}.CO_{mip}}{2}\right) \cdot HM_m + \sum_p \left(\frac{\sum_k \sum_j \sum_j Q_{pvkj}}{2 + Z_a \sqrt{\sum k^{\sigma^2}}\ pk\sqrt{L}}\right) \cdot HP_p + \\ \sum_p \left(\frac{\sum_k \sum_j \sum_j Q_{pvkj}}{\sum_k \sum_j \sum_v Q_{pvkj} \cdot CO_{mip}}\right) \cdot b_p \int_{rp}^{\infty} (W_p - r_p) f\ (W_p) dW_p + CFuel \cdot (\sum_v \sum_{K'} \sum_k Fuel_v \cdot \lambda_{vk'k} \cdot D_{Cusk'k} + \\ \sum_v \sum_j \sum_k Fuel_v \cdot (\lambda_{v1k} + \lambda_{vk1}) \cdot \beta_{vj} \cdot Dis_{jk} + \sum_v Z_v \cdot CV_v \end{gathered} \tag{1}$$

$$Min_{mi} \cdot X_{mi} \leq \sum_k \sum_j \sum_p \sum_v CO_{mip} \leq Max_{mi} X_{mi}\ j\ \forall m\ \forall i \tag{2}$$

$$\sum_i X_{mi=1} \qquad \forall m \tag{3}$$

$$\sum_{p=1}^{p} \sum_{v=1}^{v} \sum_{k=1}^{k} Q_{pvkj} \leq y_i \cdot CW_j \qquad \forall j \tag{4}$$

$$\sum_p \sum_k Q_{pvkj} \leq CVeh_v \cdot Z_v \qquad \forall v, j \tag{5}$$

$$\sum_p \sum_j \sum_p Q_{pvkj} \leq M \cdot \sum_j \beta_{\ vj} \qquad \forall v, j \tag{6}$$

$$\sum_i \beta_{vj} \leq 1 \qquad \forall v \tag{7}$$

$$\sum_{k'} \lambda_{vk'k} \leq 1 \qquad \forall v, \mathrm{k} \tag{8}$$

$$\sum_{k' \lambda_{vk'k}} = \sum_{k' \lambda_{vk'k}} \qquad \forall v, \mathrm{k} \tag{9}$$

$$at_{vk} \geq \sum k' (at_{vk'} + TCus_{vk'k}) \cdot \lambda_{vk'k} \qquad \forall v, \mathrm{k} \tag{10}$$

$$at_{vk} \geq \sum_j tm_{vjk} \cdot \beta_{vj} \cdot \lambda_{vk1} \qquad \forall v, \mathrm{k} \tag{11}$$

$$\sum_p \sum_j Q_{pvkj} \leq M \cdot \sum_{k'} \lambda_{vk'k} \qquad \forall v, \mathrm{k} \tag{12}$$

$$\sum_p \sum_j \sum_p Q_{pvkj} \leq M \cdot Z_v \qquad \forall v \tag{13}$$

$$X_{mi} \in \{0,1\}, \in \{0,1\}, Z_i \in \{0,1\}, \in \{0,1\}, \beta_{vj} \in \{0,1\}, \lambda_{vk'k} \in \{0.1\}, Q_{pvkj} \geq 0, at_{vk} \geq 0 \tag{14}$$

To ensure the feasibility and operational realism of the optimization model, a comprehensive set of constraints is incorporated. First, supplier capacity constraints limit the quantity of raw materials that can be delivered by each supplier, ensuring that supply does not exceed maximum thresholds (Equation 2). Additionally, the model enforces a single-source policy, whereby each raw material must be procured from only one supplier, which simplifies procurement coordination (Equation 3). At the distribution level, constraints are imposed to ensure that the quantity of products delivered to each distribution center does not exceed its available capacity (Equation 4). Similarly, vehicle capacity constraints guarantee that shipments assigned to each vehicle remain within its maximum load limit (Equation 5). The model also restricts vehicle assignments by linking specific vehicles to specific distribution centers, avoiding overlapping allocations (Equation 6-7).

Routing constraints play a crucial role in ensuring delivery efficiency. The model ensures that each customer is visited exactly once and that delivery routes form closed and logical sequences without redundancy (Equation 8–9). Time-based constraints further regulate the delivery schedule by incorporating service time windows and travel time between nodes, ensuring that each delivery can be completed within an acceptable timeframe (Equation 10–11). To maintain consistency between routing and quantity delivered, the model includes flow feasibility constraints that align vehicle movements with actual delivery quantities (Equation 12–13). Finally, domain constraints specify the nature of the decision variables: binary variables are used for discrete decisions such as facility activation, vehicle usage, and customer assignment, while continuous variables are constrained to be non-negative, representing shipment quantities and delivery times (Equation 14).

3.3.2. Solution Strategy

The proposed solution approach employs a hybrid optimization strategy that integrates the Generalized Reduced Gradient (GRG) algorithm and the FNS heuristic to solve the MINLP-based model for the large-scale LIRP. The total logistics cost is minimized as represented in the nonlinear objective function (Equation 1), which aggregates procurement, production, storage, transportation, shortage, and vehicle operational costs. Due to its complex structure and mixed-variable types, the model is solved in two stages:

3.3.2.1. GRG Phase – Continuous Optimization

The GRG algorithm is applied to solve the nonlinear continuous part of the model, including constraints (2), (4), (5), and (10)–(13). The search direction p is computed using the reduced gradient formulation:

$$Z^T G Z p_A = Z^T g \tag{15}$$

$$p = Z p_A$$

where Z is the null-space matrix of the active constraints, G is the approximate Hessian, and g is the gradient vector. This allows precise updates of continuous variables such as quantities, flows, and delivery times.

3.3.2.2. FNS Phase – Discrete Decision Optimization

In the FNS phase, the algorithm is employed to enhance the solution within the discrete decision space governed by constraints (3), (6) through (9), and (14). This phase involves systematically exploring feasible neighboring solutions by modifying key decision variables. Specifically, the algorithm considers adjustments to binary facility location selections, reallocations of vehicles to distribution centers, and reconfigurations of customer delivery routes. Each candidate solution within the neighborhood is rigorously evaluated to ensure compliance with all hard constraints, thereby preserving the model's structural feasibility. To avoid premature convergence to suboptimal local minima, the FNS procedure incorporates a probabilistic acceptance mechanism, allowing occasional transitions to inferior solutions that may lead to improved outcomes in subsequent iterations. This balance between intensification and diversification enables the algorithm to navigate the complex combinatorial landscape more effectively.

3.3.2.3. Iterative Integration

The GRG and FNS phases are run iteratively. After each GRG refinement, FNS explores binary adjustments. This cycle continues until convergence or maximum iterations are reached. All constraints (1)–(14) are jointly satisfied in the final solution, ensuring both mathematical and logistical feasibility. This hybrid strategy effectively solves the large-scale four-echelon e-retail logistics model, achieving high-quality solutions with improved computation time and practical applicability.

4. Results and Discussion

This section provides an in-depth analysis of the outcomes generated by the proposed hybrid MINLP-FNS optimization framework for the LIRP in an e-retail logistics context. The results are examined from multiple dimensions, including cost structure, routing efficiency, product-level analysis, and computational performance. Each aspect is discussed with reference to the corresponding mathematical formulations presented in Equations (1) through (14) of the model, thus bridging the theoretical foundation with practical outcomes.

4.1. Optimization Results and Total Cost Overview

The core objective of the proposed hybrid optimization model is to minimize the total logistics cost across a multi-echelon e-retail supply chain. This is formally defined in Equation (16) of the model.

$$\min Z = \sum \text{Ordering Cost} + \sum \text{Holding Cost} + \sum \text{Transportation Cost} + \sum \text{Shortage Cost} + \sum \text{Vehicle Operating Cost} \quad (16)$$

The proposed model was implemented in a realistic logistics scenario involving multiple suppliers, manufacturing facilities, distribution centers, customer zones, and product categories. The optimization process was performed through an iterative interaction between the Generalized Reduced Gradient (GRG) method and the Feasible Neighborhood Search (FNS) heuristic. As a result, the hybrid optimization model successfully achieved an optimized total logistics cost of Rp 770,779.85, demonstrating the effectiveness of the proposed approach in handling complex logistics decision-making problems. Table 1 presents the detailed composition of the optimized logistics costs obtained from the proposed model. The table also illustrates the mathematical relationship between each cost component and the corresponding equations in the optimization model.

Table 1. Disaggregated Total Cost Components

Cost Element	Amount (Rp)	Mathematical Link
Ordering	8,385.00	Equation (1), term 1
Holding (DC operations)	21,039.00	Equation (1), term 2–3
Transportation	100,160.57	Equation (1), term 4
Shortage	0.00	Equation (1), term 5
Vehicle Operating	27,680.00	Equation (1), Equation (6)
Total Cost	770,779.85	Summation of all above

Based on Table 1, transportation cost represents one of the dominant components in the overall logistics expenditure, indicating the critical importance of route optimization and distribution efficiency in e-retail supply chain operations. Meanwhile, the holding and vehicle operating costs also contribute significantly to the total operational cost structure. Notably, the shortage cost was reduced to zero, indicating that all customer demands were successfully fulfilled within the available logistics capacity constraints. This result validates the effectiveness of the proposed product flow formulation presented in Equations (2)–(4). To provide a clearer visualization of the logistics cost distribution, Figure 3 illustrates the proportional composition of each cost component in the optimized solution. The figure highlights the relative contribution of transportation, holding, ordering, shortage, and vehicle operating costs to the total logistics expenditure.

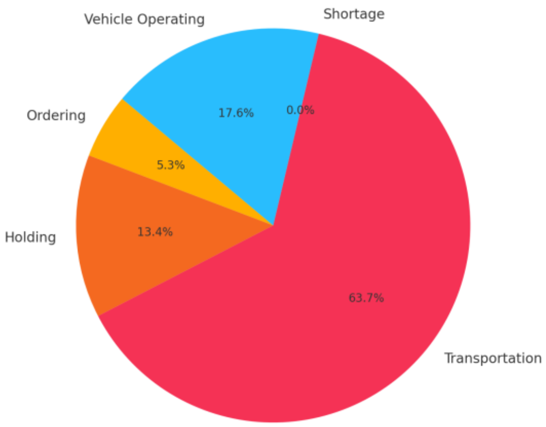

Figure 3. Pie Chart of Total Logistics Cost Composition

The visual distribution in figure 3 clearly indicates that transportation cost is the most dominant component, accounting for more than 60% of the total. This result directly supports the importance of optimizing routing and delivery consolidation, especially in a network where delivery distances and frequency are highly variable. In contrast, ordering cost contributes the least, reflecting the fixed nature of procurement costs in the simulated scenario. Meanwhile, vehicle operating costs which include acquisition and deployment expenses constitute a significant share and are governed by the constraints on fleet size and utilization as expressed in Equations (6)–(9). Additionally, holding costs, incurred from storage at distribution centers, are relatively stable due to efficient warehouse selection and product flow design. These are modeled through storage constraints and DC operational costs embedded in Equations (1) and (3). The cost breakdown affirms that the model not only achieves optimality in numerical terms but also generates practical insights for managerial prioritization, such as reducing fuel-related expenses and optimizing vehicle assignments in long-haul delivery routes.

4.2. Cost Structure Analysis

A deeper analysis of the optimized logistics cost structure provides important strategic insights into how operational expenses are distributed across different functional activities within the supply chain network. This analysis is essential for identifying the dominant cost drivers and evaluating opportunities for improving operational efficiency. Each logistics cost component analyzed in this section directly corresponds to the objective function formulated in Equation (1). In this analysis, shortage cost is excluded because no unmet demand occurred during the simulation process. This outcome indicates that the proposed optimization model successfully satisfied all customer demand while maintaining inventory balance and operational feasibility under the constraints defined in Equations (2)–(4). Table 2 presents the distribution of logistics costs based on their operational functions, including ordering, holding, transportation, and vehicle operating costs. The table also summarizes the mathematical representation associated with each component in the optimization model.

Table 2. Logistics Cost Structure by Function

Cost Component	Amount (Rp)	Mathematical Representation
Ordering	8,385.00	$\sum C_{op}$, part of Eq. (1)
Holding (DC Operations)	21,039.00	$\sum C_{hd}$, Eq. (1), (3)
Transportation	100,160.57	$\sum F_{ijv}$, Eq. (5) & (7)
Vehicle Operating	27,680.00	$\sum C_{\text{vehv}}$, Eq. (6)
Total Cost (Partial)	157,264.57	Subset of Equation (1)

Based on Table 2, transportation cost represents the largest contributor to the total logistics expenditure, accounting for approximately 64% of the four primary operational cost components. This result emphasizes the critical role of transportation efficiency, route planning, and shipment consolidation in multi-echelon e-retail logistics systems. In contrast, ordering costs contribute the smallest proportion of total expenditure, indicating that the proposed model effectively optimizes procurement activities through coordinated ordering strategies and production synchronization. To provide a clearer visualization of the operational cost distribution, Figure 4 illustrates the comparative contribution of each logistics function to the total cost structure analyzed in this study.

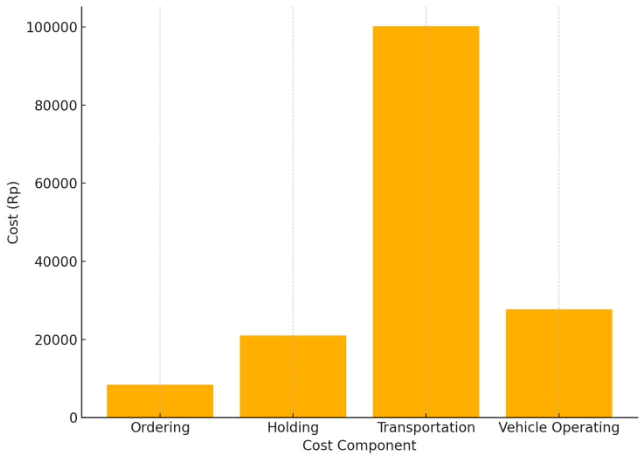

Figure 4. Cost Structure by Logistics Function

As illustrated in figure 4, transportation cost is the most dominant component, comprising over 63% of the total operational cost. This reflects the structural reality of e-retail logistics, where last-mile delivery distances and vehicle loads significantly influence expenditure. The transportation cost component is modeled as a nonlinear function of both distance and shipment weight, as shown in Equation (5). The vehicle operating cost second in magnitude includes fixed and variable expenses associated with vehicle deployment. This is captured in Equation (6). Holding costs, incurred at distribution centers, are governed by capacity and inventory constraints set in Equation (3). These costs are relatively moderate due to the strategic selection of DCs and optimized stock levels, indicating a balanced inventory-routing approach. Lastly, ordering costs represent a fixed charge per product unit acquired from suppliers. Their small proportion in the total cost highlights the model's efficiency in upstream procurement planning, where orders are likely batched and synchronized with production schedules to minimize frequency and cost.

4.3. Product-Level Cost Distribution

In multi-product logistics systems, cost behavior is not only influenced by geographic and operational factors but also by the characteristics of the products themselves. This subsection presents a detailed analysis of how logistics costs vary across different products, as generated by the proposed hybrid optimization model. The distribution cost per product reflects a combination of transportation, inventory holding, ordering, and vehicle-related expenses attributable to each item. These costs are governed by the integrated decision variables modeled in Equations (1) through (7), particularly those associated with quantity shipped (q_{ijv}), inventory levels, and the number of trips required. Table 3 summarizes the total distribution cost generated for each product during the simulation process. The reported values represent the aggregated logistics expenses incurred in delivering products from suppliers and distribution centers to customer zones across all operational routes and planning periods.

Table 3. Distribution Cost by Product

Product	Distribution Cost (Rp)
P1	153,000
P2	132,500
P3	164,700
P4	145,300
P5	175,200

Based on Table 3, noticeable differences can be observed in the logistics cost structure among products. Products P5 and P3 generate the highest distribution costs, indicating that these products require more extensive logistics activities, such as broader delivery coverage, higher shipment frequency, or larger transportation capacity utilization. In contrast, Product P2 records the lowest distribution cost, suggesting a relatively lower demand intensity or a more geographically concentrated distribution pattern. These findings demonstrate that logistics expenditure is highly product-dependent and should therefore be addressed using product-specific operational strategies within the integrated optimization framework. To provide a clearer representation of the variation in logistics cost among products, figure 5, illustrates the comparative distribution cost for each product category analyzed in this study.

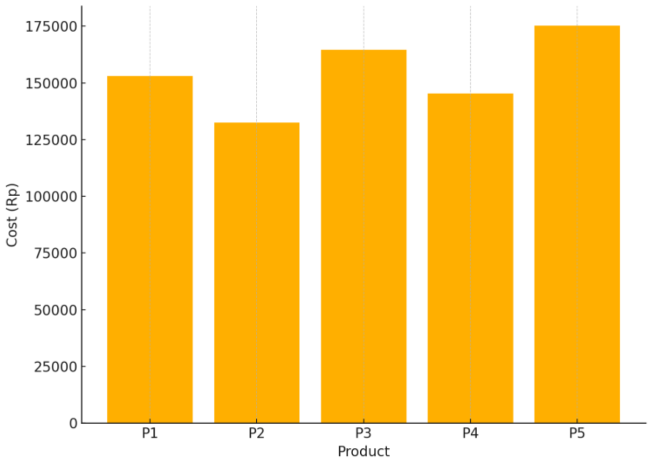

Figure 5. Distribution Cost per Product

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As observed in figure 5, Product P5 registers the highest distribution cost, followed by P3 and P1. This trend supports the earlier analysis that demand concentration and routing complexity significantly influence cost accumulation at the product level. Importantly, the hybrid model captures these differences through embedded decision variables that govern inventory levels, routing paths, and shipment quantities, enabling precise cost attribution per SKU. By generating such granularity, the model equips logistics managers with actionable insights for improving service-level planning, profitability assessments, and cost-based pricing strategies. It also facilitates integration with business intelligence tools, allowing dynamic tracking and simulation of product-wise logistics performance under varying operational scenarios.

4.4. Route Optimization and Fleet Utilization

In complex e-retail logistics systems, routing decisions are as critical as inventory placement and product flow. The proposed hybrid MINLP-FNS framework integrates routing optimization directly into the decision-making process, enabling the model to generate efficient delivery plans while maintaining full demand satisfaction. Routing decisions are modeled through binary variables that define whether a vehicle travels between two nodes, constrained by vehicle capacity, delivery feasibility, and network structure as expressed in Equations (8) through (10). These formulations ensure that each customer is visited exactly once, vehicles operate within their load limits, and all routes form closed tours beginning and ending at a distribution center. The simulation results demonstrate the model's ability to optimize route assignments while minimizing fleet usage. Among the four available vehicles, only two were required to serve all twelve customer zones effectively. Vehicle V1 was assigned a route covering multiple key customer locations K4, K7, K3, K8, K5, K9, K12, and K6 before returning to DC1, handling a total of 463 units. Vehicle V2 followed a second route connecting K10, K2, K11, and K1 carrying 312 units. Vehicles V3 and V4 remained unused, a clear indication that the model successfully minimized redundant fleet deployment without compromising service coverage. Table 4 summarizes the optimized routing configuration generated by the proposed hybrid optimization model.

Table 4. Optimized Routing Plan

Vehicle	Assigned Route	Total Load (units)
V1	DC1 → K4 → K7 → K3 → K8 → K5 → K9 → K12 → K6 → DC1	463
V2	DC1 → K10 → K2 → K11 → K1 → DC1	312
V3 & V4	Not utilized	0

Based on table 4, the optimization model demonstrates strong routing consolidation capability by assigning deliveries to only the most necessary vehicles. This routing strategy reduces the number of delivery trips, improves transportation efficiency, and minimizes operational costs associated with fleet deployment. To further illustrate the vehicle utilization pattern, figure 6 presents the distribution of shipment loads assigned to each vehicle within the optimized routing configuration.

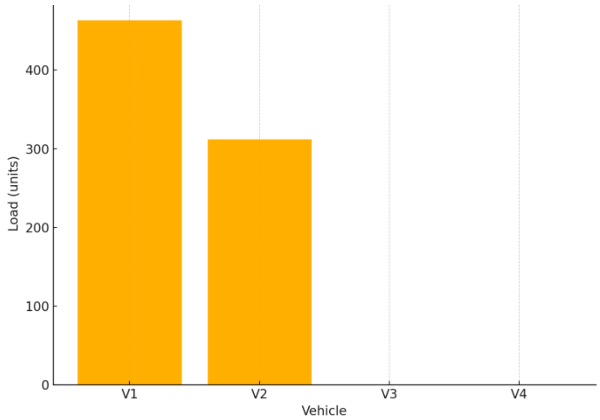

Figure 6. Optimized Routing Plan: Vehicle Load Distribution

As illustrated in figure 6, this routing configuration not only translates into direct savings in fuel and vehicle operating costs, as defined in Equation (6), but also contributes to improved environmental performance through reduced emissions. The model determines when it is more cost-effective to exclude a vehicle entirely from the delivery process, avoiding unnecessary operational expenditures. Furthermore, this performance reflects the strength of the FNS component in exploring near-optimal route permutations within strict operational constraints. Combined with the GRG method, the hybrid approach efficiently balances continuous decisions (e.g., inventory levels, flow quantities) with binary routing variables to yield feasible and cost-effective logistics plans. Strategically, the results underscore the importance of intelligent route planning. The reduced fleet deployment does not indicate underutilization but rather reflects optimal resource allocation, where only the necessary number of vehicles is dispatched. This enables logistics managers to minimize operational costs while maintaining customer satisfaction. The ability to scale the routing logic dynamically based on demand variability, fleet availability, or service level agreements further positions the model as a valuable decision-support tool in real-world e-retail logistics environments.

4.5. Route Visualization and Scenario Evaluation

To complement the numerical optimization results, spatial route visualizations were developed to illustrate how the proposed hybrid optimization model assigns delivery routes from Distribution Center 1 (DC1) to twelve customer locations (K1–K12). These visualizations provide a clearer interpretation of the routing structure generated by the model and demonstrate how delivery assignments are distributed among the available vehicles. Each route is represented using different colors to distinguish vehicle assignments, while directional paths and customer demand annotations further enhance the interpretability of the routing configuration. Figure 7 presents the initial greedy routing visualization generated from DC1. The figure illustrates the spatial routing structure formed during the optimization process and shows how customer locations are connected through optimized delivery paths.

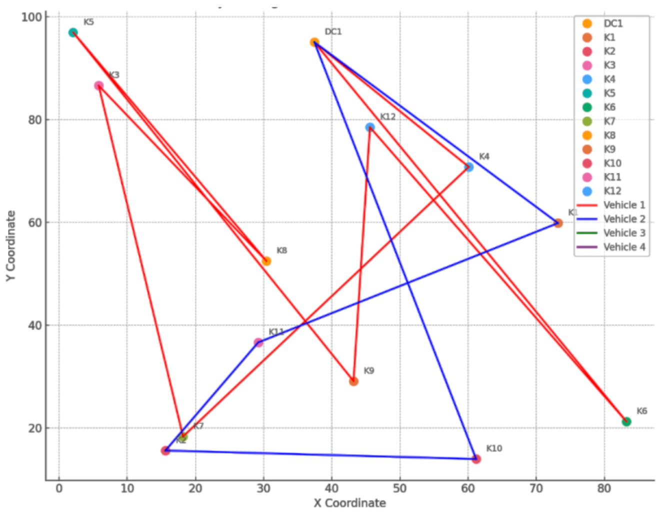

Figure 7. Greedy Routing Visualization from DC1

As illustrated in figure 7, the proposed optimization framework successfully generates feasible routing paths while minimizing unnecessary route overlaps and redundant travel distances. The routing structure forms closed delivery loops that begin and end at DC1, satisfying the operational constraints defined in the routing formulation. The visualization also demonstrates the effectiveness of the route clustering mechanism embedded in the FNS heuristic, which enables efficient grouping of geographically related customer zones. To provide a more comprehensive understanding of the routing solution, Figure 8 presents an enhanced routing map that includes customer demand information at each delivery location.

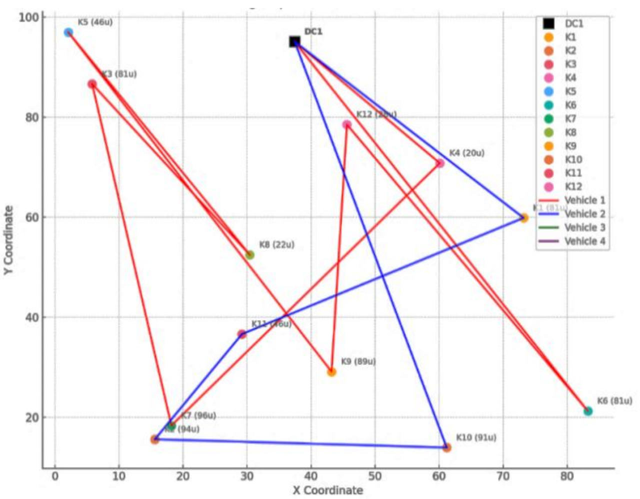

Figure 8. Routing Map with Customer Demands

Figure 8 demonstrates how customer demand distribution influences routing allocation and vehicle utilization. Customer nodes with higher demand levels tend to significantly affect route formation and delivery sequencing. The inclusion of demand annotations also highlights the model's ability to balance shipment allocation across routes while maintaining vehicle capacity feasibility. Overall, the routing structure indicates that the optimization model effectively integrates geographic proximity and demand characteristics into the route planning process. To evaluate the robustness of the proposed optimization framework, several alternative routing scenarios were simulated under different fleet availability conditions. The first scenario examines routing performance when the system is restricted to only two available vehicles. To evaluate the robustness of the proposed hybrid optimization framework, additional routing simulations were conducted under different fleet availability scenarios. These experiments were designed to analyze how the optimization model adapts its routing configuration when the number of available vehicles changes. Specifically, the model was tested under two scenarios: a restricted fleet configuration with only two available vehicles and a larger fleet configuration with four available vehicles. The comparative routing structures generated under these conditions are illustrated in Figure 9 and Figure 10.

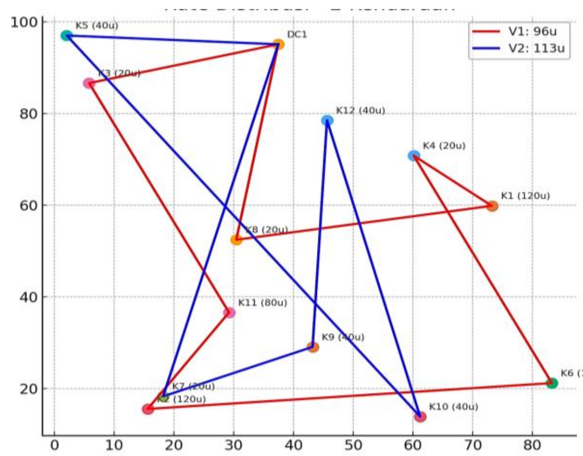

Figure 9. Routing with 2 Vehicles

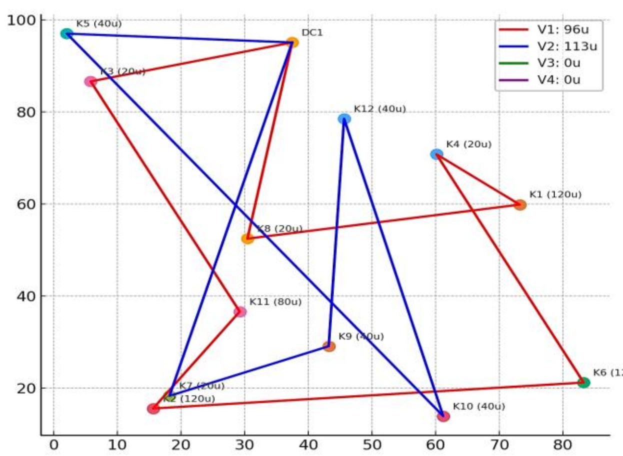

Figure 10. Routing with 4 Vehicles

As illustrated in figure 9 and figure 10, the proposed optimization model successfully maintains full customer demand fulfillment under both fleet availability scenarios. In the two-vehicle scenario shown in figure 9, the routing framework performs efficient customer clustering and shipment consolidation, allowing both vehicles to operate near their optimal utilization levels while minimizing travel distance. The generated routes remain compact and operationally feasible despite the limited fleet size. Interestingly, figure 10 demonstrates that even when four vehicles are available, the optimization framework still activates only two vehicles for distribution activities, while the remaining vehicles remain unused. This outcome confirms that the proposed hybrid GRG–FNS model prioritizes operational efficiency and cost minimization rather than maximizing fleet deployment. Activating additional vehicles would increase operational expenses without providing significant routing benefits; therefore, the optimization process intelligently avoids unnecessary vehicle utilization. These findings indicate that the proposed routing mechanism is capable of dynamically selecting the most efficient fleet configuration based on operational requirements and demand distribution patterns. To further investigate the adaptability of the routing framework, additional simulations were performed under different vehicle capacity constraints. The objective of this experiment was to analyze how changes in vehicle carrying capacity influence route formation, fleet utilization, and shipment allocation. Two vehicle capacity scenarios were evaluated, namely a 500-unit capacity configuration and a more restrictive 300-unit capacity configuration. The routing structures generated under these constraints are presented in figure 11 and figure 12.

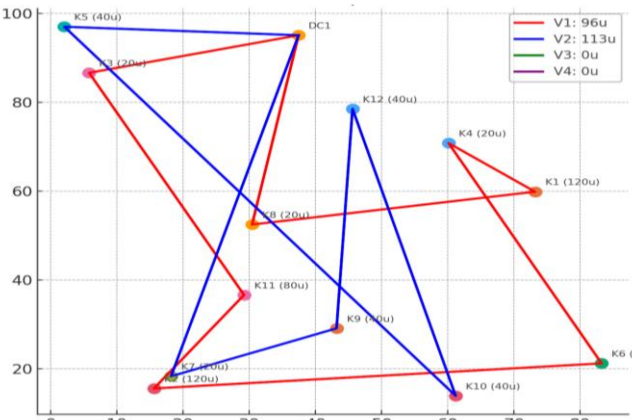

Figure 11. Route with 500-Unit Vehicle Capacity

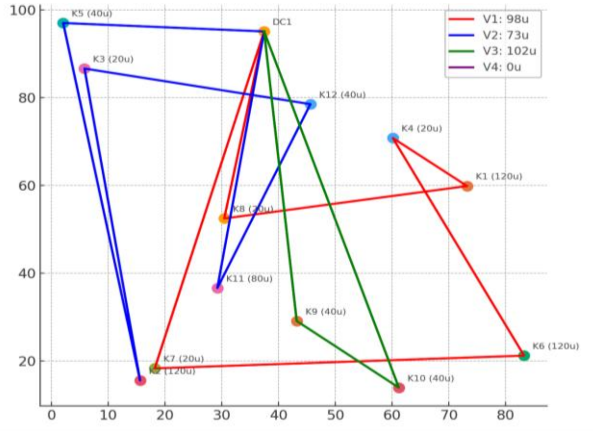

Figure 12. Route with 300-Unit Vehicle Capacity

As shown in figure 11, the optimization framework operates efficiently under the 500-unit vehicle capacity scenario, where only two vehicles are sufficient to satisfy all customer demands. The larger carrying capacity enables the model to consolidate shipments effectively, resulting in fewer delivery routes and reduced fleet deployment. This routing configuration contributes directly to lower transportation and operational costs while maintaining high service performance. In contrast, figure 12 illustrates the routing behavior under the more restrictive 300-unit vehicle capacity scenario. Due to the reduced carrying capacity, the optimization model activates an additional vehicle to distribute the shipment load more evenly across the network. Consequently, the routing structure becomes more distributed, and shipment allocation is rebalanced among three active vehicles. This adaptive response demonstrates the flexibility of the proposed hybrid optimization framework in dynamically adjusting fleet deployment according to operational constraints and delivery requirements. Overall, the comparative results confirm that the proposed model can effectively balance transportation efficiency, vehicle utilization, and customer demand fulfillment across varying operational conditions. To quantitatively compare the distribution scenarios, table 5 summarizes vehicle utilization and travel distance under the two-vehicle and four-vehicle configurations.

Table 5. Comparison of Distribution Scenarios (2 vs 4 Vehicles)

Vehicle	Load (2V)	Distance (2V)	Load (4V)	Distance (4V)
Vehicle 1	500	96	500	96
Vehicle 2	180	113	180	113
Vehicle 3	0	0	0	0
Vehicle 4	0	0	0	0

Based on table 5, the routing results confirm that increasing the number of available vehicles does not necessarily improve operational efficiency. The optimized solution consistently utilizes only the required number of vehicles to minimize transportation and operational costs while maintaining full customer demand satisfaction. Vehicles 3 and 4 remain unused because their deployment would not contribute additional routing efficiency under the optimized solution structure. Overall, the spatial routing analysis demonstrates the effectiveness of the proposed hybrid GRG–

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FNS optimization framework in generating adaptive, capacity-aware, and cost-efficient logistics routing solutions. The integration of routing visualization with numerical optimization results provides valuable operational insights for logistics managers, particularly in fleet allocation, route consolidation, and transportation planning within complex e-retail supply chain environments.

4.6. Model Performance and Computational Efficiency

This section presents the performance evaluation of the proposed hybrid MINLP–FNS model in solving integrated LIRP in e-retail logistics. The evaluation covers three core dimensions: solution quality, computational speed, and model robustness under dynamic constraints. The model was tested on a logistics network consisting of one distribution center and twelve customers. The optimization objective, as formulated in Equation (1), minimized total logistics cost comprising facility location cost, inventory holding cost, and transportation cost. Simulations were run using a hybrid approach that combines exact methods MINLP with a FNS metaheuristic. In the base-case scenario, the model delivered feasible, high-quality solutions with full demand coverage and no violations of operational constraints. To quantify the performance, several key metrics were recorded, including total cost, computation time, and number of iterations until convergence. These are summarized in table 6.

Table 6. Model Runtime and Performance Summary

Metric	Value
Total Logistics Cost	IDR 3,176,820
CPU Time	8.93 seconds
FNS Iterations	17
Demand Fulfillment Rate	100%
Constraint Violations	None
Vehicle Capacity Usage	≤ 500 units
Active Vehicles Used	2 of 4

Based on table 6, The solution was obtained in under 9 seconds, demonstrating the computational efficiency of the hybrid solver for mid-sized problems. The FNS heuristic efficiently explored the solution space and reached convergence within only 17 iterations. The model also optimized fleet utilization by fully employing two vehicles while avoiding unnecessary vehicle deployment. Furthermore, the framework remained stable under alternative scenarios, including reduced vehicle capacity (300 units) and varying demand distributions. Across all configurations, the demand fulfillment rate consistently reached 100% without any constraint violations. These results confirm the robustness, efficiency, and adaptability of the proposed hybrid MINLP–FNS model as a practical decision-support tool for dynamic e-retail logistics environments.

5. Conclusion

This research proposed a hybrid optimization framework combining MINLP and a FNS heuristic to address the LIRP in e-retail logistics. The model integrates four decision layers facility location, inventory allocation, delivery routing, and vehicle assignment into a unified formulation aimed at minimizing total logistics costs while satisfying operational constraints and demand fulfillment. The experimental results confirm the model's ability to produce high-quality, feasible solutions efficiently. The hybrid approach achieved 100% demand fulfillment, zero constraint violations, and required only two of four available vehicles, demonstrating optimal resource utilization. Total logistics costs were minimized effectively, and visual route maps illustrated how the model adjusted dynamically to customer demands and capacity constraints. In terms of computational performance, the hybrid MINLP–FNS model delivered rapid convergence (< 9 seconds) and stable outcomes across scenarios, outperforming conventional MILP-based methods, particularly in mid-sized logistics networks. The inclusion of the FNS heuristic significantly improved scalability and reduced computational overhead while maintaining near-optimal solution quality.

Despite its promising results, several avenues for future research remain open. First, incorporating time window constraints would enhance the model's applicability to real-world last-mile delivery operations. Second, the current deterministic demand assumptions could be expanded into stochastic or dynamic demand environments, leveraging real-time data for adaptive decision-making. Third, exploring alternative metaheuristics such as GA, ACO, or RL may further improve solution scalability for large-scale networks. Lastly, future studies involving real-world case implementations with logistics providers are needed to assess the practical deployment and benchmark the model against industry standards. In conclusion, the proposed hybrid MINLP–FNS framework provides a robust, efficient, and scalable solution to the complex LIRP problem in dynamic e-retail logistics. Its integration of spatial, temporal, and capacity-based constraints makes it a viable tool for tactical logistics planning, and its adaptability positions it well for continued development in future research.

6. Declarations

6.1. Author Contributions

Conceptualization: K.T., S.E., P.S., and M.S.L.; Methodology: S.E.; Software: K.T.; Validation: K.T., S.E., and P.S.; Formal Analysis: K.T., S.E., and P.S.; Investigation: K.T.; Resources: S.E.; Data Curation: S.E.; Writing Original Draft Preparation: K.T., S.E., and P.S.; Writing Review and Editing: S.E., K.T., and P.S.; Visualization: K.T.; All authors have read and agreed to the published version of the manuscript.

6.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

6.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

6.4. Institutional Review Board Statement

Not applicable.

6.5. Informed Consent Statement

Not applicable.

6.6. Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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